Vacuum suction casting process for aluminium and copper alloy pipes

Part II—Calculation of velocity of metal surface rise and castable length

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Numerical analysis of the velocity of the molten metal's surface rise in the vacuum suction casting process was carried out. Experiments were done with water and molten aluminium.

Quantitative agreement of the calculation with the experiments was obtained for different conditions. When the mould diameter was too small, oscillation of the molten metal (or water) in the mould was generated by the inertia of the fluid flow motion.

An example of the calculation procedure for the determination of the maximum castable length of a pipe with good surface quality and uniform thickness has been shown.

1. INTRODUCTION

In the previous paper, it became clear that, in the vacuum suction casting process, the velocity of the metal surface rise in the mould should be higher than 3 cm/s for the casting of pipes with sound surface quality and uniform thickness.

Ohnaka et al. have analysed in detail that the velocity of the metal surface rise in the mould is an important factor affecting the surface quality of columnar aluminium ingot cast in metallic moulds. Heutecker has referred to effects of the velocity on the surface quality.

In the vacuum suction casting process, the velocity depends on process variables such as the volume of the reservoir and the mould and varies with time during the operation. The castable length with velocity higher than the critical value may be shorter than the maximum castable length which is limited to the molten metal pressure head corresponding to 76 cm Hg.

It is necessary to determine the relationship between the velocity and process variables for the casting of a given length pipe with good surface quality and uniform thickness.

In this paper, numerical analyses are made and the calculated results are compared with those of the experiment using water and molten aluminium.

2. DERIVATION OF EQUATIONS

A calculation model is shown in Fig. 1. A mould (volume $V_1$, inner diameter $d_1$, length $l_1$ and initial pressure $P_1$) is connected to the reservoir (volume $V$ and initial pressure $P$) through the valve $A$. Molten metal (or water) is sucked into the mould with the velocity $v$.

These symbols are summarised in Table I.

The following assumptions are made in the analysis.

1. The velocity of molten metal (or water) in the radial direction is neglected. (Piston flow)
2. The volume of the connecting part between the reservoir and the mould is negligible.
3. Kinetic momentum caused by lowering the metal surface in the crucible is negligible.
4. Velocity of the gas (air) flow from the mould to the reservoir after opening the valve $A$ is assumed to be a function of the pressure difference between the mould and the reservoir.

Before the immersion of the mould into the molten metal bath, the initial volume, pressure and temperature of the mould are $S_1$, $P_{1}$, and $T_{1}$, respectively. After the immersion of the mould the values change to $S_1$ ($l_1 - l_0$), $P_{1}$ and $T_{1}$, respectively.

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Fig. 1—Model for calculation
TABLE I List of Symbols

\[ a = \text{constant, Eq. (32)} \]
\[ A = h + h + P_\text{a}, \text{Eq. (5)} \]
\[ B = 4 \left( \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \right) \left( \frac{h_0}{y_1} \right) \left( 1 - \frac{y_1^2 + 273}{y_1 + 273} \right), \text{Eq. (6)} \]
\[ d_\text{h} = \text{inner diameter of the mould} \]
\[ d_\text{c} = \text{inner diameter of the crucible} \]
\[ f(t) = \text{heated rate of air in the mould, Eq. (14)} \]
\[ g = \text{acceleration of gravity} \]
\[ h_0 = \text{initial height of the molten metal in the mould} \]
\[ h_1 = \text{height of the molten metal in the mould} \]
\[ h_\text{c} = \text{descending distance of the metal in the crucible} \]
\[ h_\text{f} = \text{final suction height} \]
\[ K = \text{flowability coefficient of air, Eq. (19)} \]
\[ K_\text{a} = \text{constant, Eq. (32)} \]
\[ l_0 = \text{length of the mould} \]
\[ l_\text{h} = \text{initial immersed length of the mould} \]
\[ n = \text{quantity of air in the reservoir at P, V and T} \]
\[ m = \text{quantity of air in the mould at P, V and T} \]
\[ P = \text{pressure in the reservoir} \]
\[ P_\text{a} = \text{initial pressure in the reservoir} \]
\[ P_\text{f} = \text{final pressure in the reservoir and the mould} \]
\[ P_\text{c} = \text{atmosphere pressure} \]
\[ P_1 = \text{pressure in the mould} \]
\[ P_1^\text{m} = \text{initial pressure in the mould after immersion} \]
\[ R = \text{gas constant} \]
\[ Re = \text{Rayleigh number } = v d/s \]
\[ S_\text{a} = \text{cross section area of the mould} \]
\[ S_\text{c} = \text{cross section area of the crucible} \]
\[ t = \text{time after suction} \]
\[ T = \text{air temperature in the reservoir} \]
\[ T_\text{c} = \text{air temperature in the reservoir} \]
\[ T_\text{a} = \text{initial air temperature in the reservoir} \]
\[ v = \text{rising velocity of the molten metal surface} \]
\[ V = \text{reservoir volume} \]
\[ V_\text{c} = \text{mould cavity volume} \]
\[ y_1 = \text{specific gravity of the molten metal (or water)} \]
\[ z = \text{friction factor for the mould surface} \]
\[ \nu = \text{kinematic viscosity of the molten metal (or water)} \]

Subscript:

- \( \text{m} \) = divided value of pressure by specific gravity of mercury (i.e., pressure represented by mercury head)

By considering the balance of force, initial pressure difference in the mould after the immersion \( P_\text{f} \) is expressed by:

\[ P_\text{f}^\text{m} = P_\text{a} + \left( h_1 - h_0 \right) y_1 \]  

From the equation of state:

\[ P_\text{f}^\text{m} = \frac{P_\text{a}^2 S_1}{R(T_\text{a} + 273)} = \frac{P_\text{a}^2 S_1}{R(T_1^0 + 273)} \]  

From Eq. (1) and Eq. (2), we can obtain:

\[ h_0 = \frac{1}{2} \left( \frac{h_1 - h_0}{L} \right) \]  

\[ P_\text{f}^\text{m} = P_\text{a} + \left( h_1 - \frac{1}{2} (L - h_1^2 - h_1) \right) y_1 \]  

where

\[ L = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{y_1} \]
\[ B = 4 \left( \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \right) \left( \frac{h_0}{y_1} \right) \left( 1 - \frac{y_1^2 + 273}{y_1 + 273} \right) \]

2. \( v \) and \( h_1 \)

From the law of conservation of mass of gas after valve \( A \) is opened;

\[ \frac{\nu}{T_\text{a} + 273} + \frac{P_1^\text{m}}{T_1 + 273} = \text{const.} \]  

Equation (7) is differentiated by time:

\[ \frac{\nu}{T_\text{a} + 273} = \frac{P_1^\text{m}}{T_1 + 273} + \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} \]  

\[ + \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} - \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} = 0 \]  

Where, \( V \) is a constant volume of the reservoir.

Hence:

\[ \frac{dv}{dt} = 0 \]  

Equation (9) is submitted to Eq. (8):

\[ \frac{\nu}{T_\text{a} + 273} = \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} \]  

\[ + \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} - \frac{P_1^\text{m}}{T_1 + 273} \cdot \frac{v}{T_1 + 273} = 0 \]  

By considering the heat balance in the reservoir,

when \( \frac{dn}{dt} \geq 0, \frac{dn}{dt} \leq 0 \),

\[ \frac{dn}{dt} = \frac{\nu}{T_\text{a} + 273} \]  

and when \( \frac{dn}{dt} \leq 0, \frac{dn}{dt} = 0 \).

Two expressions similar to Eq. (11) and Eq. (12) are now written for the temperature of air in the mould.

When \( \frac{dn}{dt} \geq 0, \frac{dn}{dt} = 0 \),

\[ \frac{dn}{dt} = f(t) \]  

and when \( \frac{dn}{dt} \leq 0, \frac{dn}{dt} = f(t) \)

where, \( f(t) \) is heated rate of air in the mould.

Usually, Eq. (11) and Eq. (14) should be satisfied, since both \( \frac{dn}{dt} \leq 0 \) and \( \frac{dn}{dt} > 0 \) will occur only in the case that the oscillation of the metal is caused.

\[ V_\text{c} \text{ is the volume of the mould cavity.} \]

Hence:

\[ V_\text{c} = \left( h_1 - h_0 \right)^2 \]  

Differentiating Eq. (15) with respect to time \( t \) yields:

\[ \frac{dv}{dt} = -S_1 \frac{dn}{dt} \]  

Where, \( \frac{dn}{dt} \) is the velocity of the rising molten metal.

\[ \frac{dn}{dt} = \nu \]  

Substituting Eq. (17) in Eq. (16) provides;

\[ \frac{dv}{dt} = -S_1 \nu \]  

Now, \( \frac{dn}{dt} \) is the flow rate of the mass of gas (air).

Hence;

\[ \frac{dn}{dt} = -\left( \frac{dn}{dt} \right) = k \]  

Substituting Eq. (19) in Eq. (11) yields;

\[ \frac{dn}{dt} = k R \left( T_\text{a} - T \right) \frac{v}{T_\text{a} + 273} \]  

Substituting Eq. (11)', Eq. (14) and Eq. (18) in Eq. (10) yields;

\[ \frac{v}{T_\text{a} + 273} = \frac{\nu}{T_\text{a} + 273} \]  

\[ + \frac{P_1^\text{m}}{T_1 + 273} \cdot f(t) \]  

Combining Eq. (19) and Eq. (11) yields;

\[ \frac{dn}{dt} = k R \frac{v}{T_\text{a} + 273} \]  

By substituting Eq. (21) and Eq. (15) in Eq. (20) and rearranging, we have;
\[
\begin{align*}
\frac{dP_1}{dt} & = \left( \frac{P_1}{T_1+273} \right) \frac{f(t)+K}{Y_1} \left( \frac{P_0-h_0}{T_1+273} \right) \frac{S_1}{Y_1} \\
\text{Now, the equation of motion in this system at pressure } P_1 \text{ in the mould can be expressed as follows:} \\
P_a = P_1 + \gamma h_1 + \gamma \frac{P_1}{T_1} + \frac{1}{2} \frac{\gamma \kappa}{\gamma - 1} \frac{dP_1}{dt} + \gamma v^2 \\
\text{The second and third term of the right side of Eq. (23) correspond to the metal head, and the fourth term is a dynamic pressure and friction loss term and the fifth term is an inertia of motion. The following expression between } h \text{ and } h_1 \text{ is given from the mass balance:} \\
(h_1-h_0) \frac{S_1}{S_2} = h_2 \frac{S_1}{S_3} \\
\text{Combining Eq. (23) and Eq. (24) and rearranging:} \\
\frac{dv}{dt} = \frac{\gamma (v_0 - h_1)}{Y_1} \left( \frac{P_1 - P_0 - \gamma h_1 + \gamma \frac{P_1}{T_1} + \frac{1}{2} \frac{\gamma \kappa}{\gamma - 1} \frac{dP_1}{dt} \gamma v^2}{\gamma - 1} \right) \\
\text{Equation (14), Eq. (11'), Eq. (21), Eq. (22) and Eq. (25) are numerically integrated under the following initial conditions using the Runge-Kutta-Gill method:} \\
T_1 = T_{1,0}, \quad P = P_0, \quad v = 0, \quad h_2 = h_0 \\
\text{For the friction factor in the mould, the value in a smooth pipe is employed:} \\
\lambda = C_1 \left( \frac{v}{D} \right) \frac{C_2}{C_0} \\
\text{Re} \geq 2230 \text{, } \lambda = 64 / \text{Re} \text{, } \lambda = 0.31644 / \text{Re} \\
\text{Final pipe length is obtained under the conditions of the velocity } v = 0, \text{ pressure } P_1 = P_1, \text{ and } dv/dt = 0. \\
\text{Substituting these conditions in Eq. (25):} \\
(P_1 - P_0) - \gamma h_1 + \gamma \left( \frac{h_1 - h_0}{S_2} \right) S_1 = 0 \\
\text{From the conservation of the total mass of the air at the initial and the final state:} \\
P_1 \left( \frac{v}{T_1+273} \right) + \left( \frac{\gamma \kappa}{\gamma - 1} \frac{dP_1}{dt} \right) = P_0 \left( \frac{S_2}{T_1+273} \right) + \frac{\gamma \kappa}{\gamma - 1} \frac{dP_1}{dt} \\
\text{Thus, } h_1 \text{ is obtained by combining Eq. (28) and Eq. (29):} \\
-h_1 \left( \frac{T_1+273}{k} \right) - \frac{v}{T_1+273} \\
\text{3. DECISION OF VALUE OF } K \\
The value of } K \text{ is very important for the evaluation of the velocity and depends on the flow resistance of the valve } B. \text{ The value of } K \text{ was obtained by the experiment where the bottom end of the mould was sealed and the pressure change in the mould was measured after opening the valve } B. \text{ Under this condition, } dv/dt, v \text{ and } dT/dt \text{ are zero.} \\
\text{Therefore, from Eq. (22):} \\
P_{B0} + P_a S_2 S_3 + P_0 S_4 \\
\text{Where, from the assumption (4):} \\
K = K_{B0} (P_{B0} - P) \\
\text{From the conservation of the mass:} \\
P_2 = P_1 \left( \frac{S_1}{S_2} \right) \left( \frac{P_1}{P_2} \right) + \left( \frac{S_2}{S_3} \right) \left( \frac{P_1}{P_2} \right) \\
\text{The pressure } P_1 \text{ was measured by a strain gauge type pressure sensor. From Eq. (30), the variation of } K \text{ with } P_1 \text{ was calculated. These results are shown in Fig. 2. This curve is described approximately as:} \\
K = 0.00548 (P_1 - P) / T_1^{3.63} \\
\text{4. EXPERIMENT WITH WATER} \\
\text{As a simple case, an experiment with water has been carried out.} \\
\text{Water was used for the following reasons.} \\
1. \text{Solidification during the suction does not occur.} \\
2. \text{Physical properties are well known.} \\
3. \text{Temperature change during the suction is negligible.} \\
\text{4.1 EXPERIMENTAL PROCEDURE} \\
\text{An acryl resin cylindrical tank (inner diameter, 30cm, depth, 30cm) was used as a crucible and filled with water to 26cm height from the bottom. Acryl resin pipes shown in Table II were used as moulds. This pipe was dipped into water and then a stop valve, } B, \text{ was open. After the motion of water stopped, the final suction height was measured under the various conditions (various pipe diameters, reservoir volumes and initial pressure differences). The}
The experimental values agree well with the calculated values. The final height increases linearly and proportionally to the initial pressure difference, and becomes higher with the smaller diameter of pipe at the constant initial pressure difference or at the constant reservoir volume; it becomes lower with the smaller reservoir volume. When the pipe diameter is too small or the initial pressure difference too large, the velocity oscillates as expected from the calculation.

6. EXPERIMENT WITH MOLten ALUMINUM

6.1 EXPERIMENTAL PROCEDURE

The experimental apparatus is the same as shown in the previous paper. Low carbon steel tubes and cast iron tube shown in Table III were used as moulds. Casting temperature was 700°C. The velocity was measured only in the cast iron mould using the apparatus shown in Fig. 3b. A sensor for measuring the velocity was composed of a two-hole insulating refractory tube, 2mm in diameter, and two parallel enamelled copper wires through the holes, and the tip of the sensor was exposed in the mould. These sensors were set at the height of 11, 18, 25, 32, 39, 46 and 53cm, respectively, from the bottom end of the mould. After the setting, the holes for the sensors were sealed by a refractory cement for avoiding the leak of air.


![Fig. 5—Variation of height of water with time (reservoir volume; 14000cm³).](image)

velocity was measured using the apparatus shown in Fig. 3a. Sensors made of copper wire, 1.5mm in diameter, were inserted into the mould cavity. When sucked water passes each of the sensors, the potential between resistance (100Ω) suddenly drops according to the increase of the circuit current. This change was recorded at the interval of 3ms. The velocity was obtained by dividing the distance between the two neighbour sensors by the time interval of the drastic change of the potential.

4.2 RESULTS AND DISCUSSIONS

The relationship between the final height and the initial pressure difference are shown in Fig. 4. The variation of the water surface position with time are shown in Fig. 5. The following physical properties were used in these calculations:

\[ v = 0.01 \text{ cm}^2/\text{s} \]

\[ \gamma = 1.0 \text{ g/cm}^3 \]

(34)

| TABLE III Moulds for Experiments with Molten Aluminium |
|-----------------|-----------------|-----------------|
| No.             | (cm)            | (cm)            |
| Material        | Low Carbon Steel| Low Carbon Steel| Cast Iron |
| Length (la)     | 63.8            | 74.7            | 20.0 |
| Inner Dia. (di) | 1.17            | 1.61            | 60   |
| Outer Dia.      | 1.51            | 2.05            | 90   |

Fig. 6—Variation of height of molten aluminium with time (calculated value).

5.2 RESULTS AND DISCUSSIONS

The calculated variation of the height with time for moulds of 11.7mm and 16.1mm diameter at the initial pressure difference of 20cmHg are shown in Fig. 6. The physical properties used for the calculation were as follows:

\[ v = 0.0058 \text{ cm}^2/\text{s} \]

\[ \gamma = 2.3 \text{ g/cm}^3 \]

(35)

In both cases, the molten metal has shown the free damping oscillation by inertia of the fluid flow motion. In experiments, final height was 15.3cm for 11.7mm diameter and 18cm for 16.1mm diameter mould. This is different from the calculation where the final height becomes higher with larger diameter of the mould. The region near the tip of the solidified metal in the mould was shown in Fig. 7. The top of the casting by the mould of 11.7mm diameter was convex and similar to that in the
flowability test using an alloy solidifying with equiaxed structure reported by Flemings et al. They have concluded that it arose from the solidification of the metal with many small particles of equiaxed crystal at the tip.

In the calculation, the solidification of the metal is not considered and it may be solidified at the position A' in Fig. 7, before it reaches the maximum position A in Fig. 6, by a similar mechanism. On the other hand, in the case of a 16.1mm diameter mould, the top of the solidified column was concave. The positions B' and C' in Fig. 7, nearly correspond to B and C in Fig. 6. Therefore, it may be considered that, complete solidification did not occur before the metal surface reached the maximum height (B in Fig. 6) and then descended to the position C, forming the thin solidified shell as shown in Fig. 6b.

This oscillation disappeared gradually by the solidification and viscosity. Thus, in the case of a small diameter, the oscillation should be taken into account. No oscillation larger than 2.5mm occurred in the calculation under the same conditions as aluminium. The effect of the initial pressure difference to the final height was obtained in Fig. 8, for the cast iron mould where the oscillation did not occur. The air in the mould may be heated by the radiation from the molten metal surface. In Fig. 8, the experimental value is compared with the calculated value under the condition where the air in the mould is not heated, or heated at the rate of 5°C/s, 10°C/s and 15°C/s, respectively. The experimental value existed within the calculated values for 5°C/s and 10°C/s. The heating effect of air in the mould varied the final height to within 5%. The variation of the height with time in the cast iron mould is shown in Fig. 9. The molten metal did not oscillate as expected from the calculation. The pressure change in the mould and the reservoir agreed well with the experimental values as shown in Fig. 10.

6. MAXIMUM CASTABLE LENGTH

In the vacuum suction casting process, the castable length is limited to the length corresponding to the maximum initial pressure difference between the mould and the reservoir, i.e., 76cmHg. However, in Part I, the authors have mentioned that the velocity of the metal surface rise should be larger than 5cm/s, at least, for a pipe with a good surface quality and also actual holding time difference (net cooled time of the metal through the mould) between the top and the bottom of the pipe should be less than 4s for the pipe of uniform thickness within 2mm of variation. Therefore, the castable length may be decided by considering these restrictions.

For example, in the case of the reservoir of 14000cm³, the mould diameter and its length of 60mm and 300cm, respectively, the final length is calculated to be 237cm from Eq. (28) and Eq. (29). Figure 11 shows the velocity v and height h with time. By considering that I should be less than 4s and v should be higher than 5cm/s, the castable length is not 237 but 160cm.
7. CONCLUSIONS
(1) Numerical analysis of the velocity of the molten metal surface rise and the castable length was carried out. Once the various process variables such as the volume of the reservoir, the mould diameter, the mould length and the initial pressure difference were determined, they could be obtained by the numerical calculation. Conversely, when the velocity satisfying the condition for a good surface quality and a uniform pipe thickness is given, the castable length and process dimensions for the length may be determined by the calculation developed in this paper.
(2) Quantitative agreement of the calculation with the experiment has been shown for a number of different conditions using water and molten aluminium.
(3) When the mould diameter is too small, the oscillation of the molten metal in the mould has been generated by the inertia of the fluid flow motion. This oscillation did not occur for the moulds of larger than 25 mm.

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REFERENCES